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Ferrofluids lubrication: a status report

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ABSTRACT

Ferrofluids (FFs) are stable colloidal suspensions composed of single-domain magnetic nanoparticles dispersed in a carrier liquid. And it is an intelligent material, exhibiting normal liquid behaviour coupled with superparamagnetic properties. Since the properties and the location of these fluids can easily be influenced by a magnetic field, FFs have recently attracted many scientific, industrial and commercial applications. Lubrication is one of the most important applications for FFs, and the advantage of FFs as lubricant, over the conventional ones, is that the former can be retained at the desired location with moderate magnetic fields. The main focus of this paper is to present a comprehensive review on FFs lubrication theories based on the three flow models of Neuringer–Rosensweig, Shliomis and Jenkins. Besides, a few experimental studies on FFs lubrication are discussed briefly. Copyright © 2015 John Wiley & Sons, Ltd.

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INTRODUCTION

Ferrofluids (FFs) are stable colloidal suspensions of magnetic particles, such as Fe_3O_4,[1] Ni-Fe,[2] Co[3] and ε-Fe_3N,[4] in a carrier liquid. Each of the particles can be treated as a permanent magnet with a magnetic moment of the order of about $10^4\,\mu_B$ (Bohr magneton).[5] Brownian motion prevents these 10 nm diameter particles from settling under gravity. Under an external magnetic field, the magnetic moment of the particle will try to align with the field direction leading to a macroscopic magnetization. A surfactant is chosen so that its molecules interact with the magnetic particles, via a bond of a functional group, to form a tightly bonded monomolecular layer around the particles, which can avoid particle agglomeration for van der Waals attraction or non-uniform external magnetic fields.[6] Liquid, as the carrier of particles, is selected to conform with its field of application. To date, a wide range of carrier liquids has been employed, such as mineral oil, hydrocarbon-based liquid and water.[7] Recently, ionic liquid has been used as carrier liquid for its negligible vapour pressure and flammability.[8–10]

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Stable suspensions of this kind were first synthesised in 1964 by Papell,[11] and the method involved wet-grinding magnetite (Fe_3O_4) in a ball mill with the presence of oleic acid in kerosene. It took about several months of continuous mill operation to produce a stable magnetic colloid. Latter, a chemical co-precipitation method[12] for the preparation of particles of magnetite, maghemite and substituted ferrites suitable for use in FFs has been the subject of many patents and publications. It is a very efficient method of production, and reaction itself proceeds rapidly. The highest saturation magnetization of the conventional magnetite-based FFs prepared by co-precipitation is about $95\,\mathrm{kA\,m^{-1}}$, which corresponds to a particle volume concentration of approximately 0.28. To achieve a higher magnetization, a novel magnetic colloid containing metallic particles was obtained through the evaporation of the ferromagnetic metals (Fe, Co) or their alloys in the presence of a surfactant and carrier liquid.[13] However, there comes an oxidation problem for the pure metallic-based FFs in the atmosphere.[14] By a new method of vapour–liquid reaction between ammonia gas and iron carbonyl, ε-Fe_3N-based FFs were synthesised,[15,16] which shows better chemical stability.[17,18]

The physical properties of FFs, in general, are determined by their base properties, dispersed phase content, aggregation stability of particles and the external magnetic field strength. One of the most salient features of the homogeneous fluids is that it can be magnetised by applying an external magnetic field, but still retaining the properties of the fluid. In FFs, individual nanoparticles are separated by the surfactant, so that they move freely more or less and the fluid equilibrium magnetization can be accurately described by the Langevin function.[6] Owing to their unique physical and chemical properties, these ferromagnetic liquids have attracted wide interest since their inception in the late 1960s. Till now, the most usual engineering applications of FFs are in sealing, grinding, separation, ink-jet printing, damper and so on.[19–22]

Lubrication is another important application for FFs. The main advantage of FFs as lubricant, over the conventional oil, is that the former can be retained at the desired location by an external magnetic field and still possesses flowability at the same time.[23] Further, with a designed magnetic field, lubricant is prevented from leaking and polluting the environment.[24] Moreover, when subjected to an external magnetic field, viscosity of the lubricant will increase, and hence, its load capacity will be improved.[25] The magnetic particles used in the carrier are much finer than the surface roughness, and they do not cause any wear on the rubbing surface.[26] As the striking features mentioned earlier, FFs lubrication is expected to be a promising supplement for traditional oil lubrication techniques.

RHEOLOGICAL PROPERTIES OF FERROFLUIDS

For FFs lubrication system, the rheological property of the fluid plays a pivotal role since it depends on the presence of a magnetic field. Under the external magnetic field, the fluid presents non-Newtonian performance that is characterised by a field-dependent yield stress and an increase of viscosity.[27] In addition, as pointed by Berkovsky *et al.*,[28] the viscosity of colloids is also influenced by temperature, interactions between particles, particle concentration, shape, dimension and viscosity of the carrier liquid. There is no general equation for the viscosity of FFs.

For isotropic diluted suspensions with spherically shaped particles, Einstein showed that the approximating relationship of viscosity of a suspension on the volume fraction, which may be represented by[29]

$$\eta = \eta_0(1 + 2.5\Phi) \tag{1}$$

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where η is the viscosity of colloid, η_0 is the viscosity of carrier liquid and Φ is the particle volume fraction.

The viscosity of concentrated suspensions can be described by the following relationship:[7]

$$\eta = \eta_0(1-\Phi)^{-5/2} \tag{2}$$

And the equation was derived by Vand in the 1940s, taken into account of the hydrodynamical interactions and the collisions between particles, but neglected Brownian motion.

However, the real FFs may differ considerably from the models. For water-based FFs, the comparison of the Einstein equation with the experimental results was carried by Hong *et al.*,[30] and it has been confirmed that the Einstein model is valid in very dilute suspension. Huang *et al.*[31] analysed the relation between the viscosity and particle concentration using ε-Fe_3N-based FFs. Similar form as Einstein equation was obtained (Equation 3) when the solid particles volume fraction was less than 0.02. The factors such as dipolar–dipolar interaction, shape anisotropy, magnetic agglomerate, chains-structure and surfactant layer contribute to the strong increase of the coefficient.

$$\eta = \eta_0(1+19.59\Phi) \tag{3}$$

Further investigation on the viscosity of concentrated suspension was carried out by Véká *et al.*[28] using capillary viscosimeter. The viscosity measurement versus volume fraction of suspension in the absence of the field is presented in Figure 1, and it is in line with the Vand's model since the particle–particle interaction is considered.

A similar study was reported by Hezaveh *et al.*[32] using a self-prepared Fe_2O_3-based FFs. Experimental results showed that viscosity increased nonlinearly with increasing particle mass fraction (Figure 2), and this phenomenon was explained due to more resistance that rose against external force as particles must move out each other way.

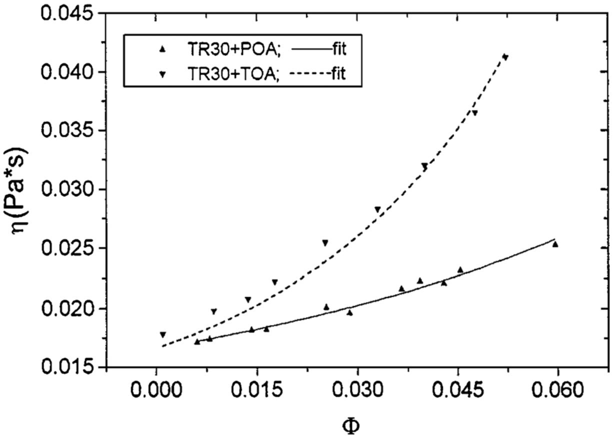

Figure 1. Viscosity versus volume fraction for FFs. The fitting curves were obtained with Vand's model.[28] Transformer oil (TR30) as a carrier liquid, pure oleic acid (POA) and technical grade oleic acid (TOA) as surfactant, respectively.

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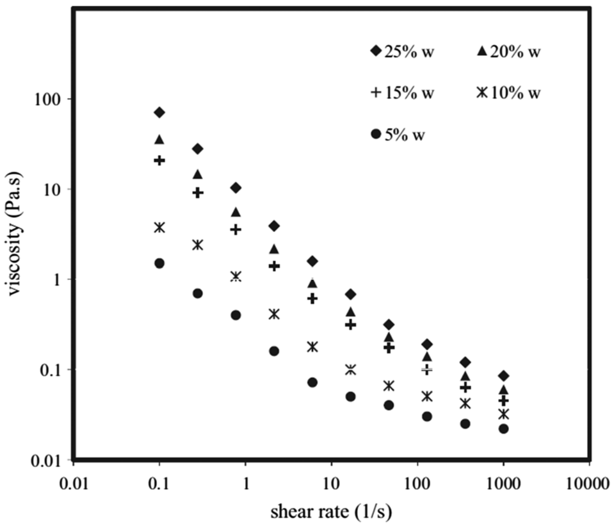

Figure 2. Relationship with viscosity and particle mass fraction.[32]

Investigation of the FFs rheological properties in the presence of magnetic field is a recurring topic. Under the external field, the rheological properties of FFs are modified, depending on their microscopical properties. Rheological experimental results, as well as theoretical studies, correlating the change of the viscosity of a sheared FFs under the influence of a magnetic field, namely the so-called magnetoviscous effect, was discussed by Pop *et al.*[33] and Masoud Hosseini *et al.*[34] It was established that the applied field had an obvious effect on the rheological properties of FFs and the magnetoviscous effect was dominated mainly by the microstructure (particle–particle interaction in the magnetic field) of the FFs.

For the commercial FFs, the dependence of the magnetoviscous effect on shear rate was reported by Odenbach *et al.*[35] as shown in Figure 3. Obvious field-dependent shear thinning was observed, leading

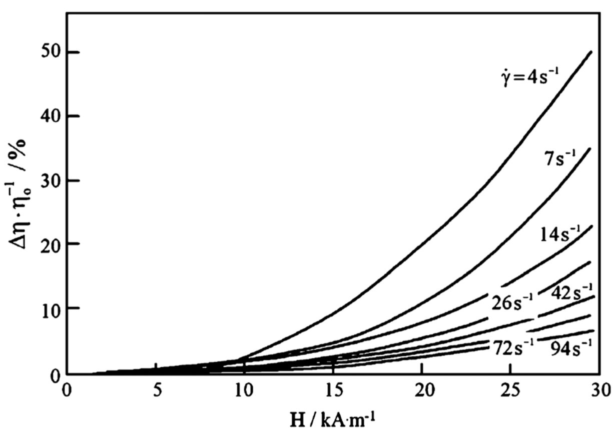

Figure 3. The magnetoviscous effect in a commercial magnetite-based ferrofluids for various shear rates.[35]

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to the assumption that the magnetic microstructures were broken by shear. Similar behaviours of shear thinning were also reported by Hong[30] and Uhlmann.[25]

FERROFLUIDS LUBRICATION BASED ON THREE FLOW MODELS

Nowadays, FFs have been successfully employed as lubricants in various hydrodynamically lubricated bearings for the controllable and positioning properties. However, due to the fact that the FFs are essentially disperse medium, the equations used for governing the flows of the FFs depend in many cases on the type of flow and on the character of the magnetic field. And among the field of FFs film lubrication, three theoretical models of Neuringer–Rosensweig,[36] Shliomis[37] and Jenkins[38] contribute to analyse the bearing characteristics. For a better understanding of FFs lubrication, the main equations for each model are briefly addressed.

The system of equations proposed by Neuringer and Rosensweig for governing the FFs is well known, and its main feature is the consideration of the magnetic body force. The equations governing the steady flow of a viscous, incompressible, magnetizable and nonconductive fluid in the presence of an applied magnetic field are as follows[36]:

$$-\nabla p + \mu_0(\mathbf{M}\cdot\nabla)\mathbf{H} + \eta\nabla^2\mathbf{v} - \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = 0 \tag{4.1}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{4.2}$$

$$\nabla\cdot(\mathbf{H} + 4\pi\mathbf{M}) = 0 \tag{4.3}$$

$$\nabla\times\mathbf{v} = 0 \tag{4.4}$$

Here, **v** and p are the fluid's velocity vector and pressure, respectively. **H** and **M** are the field of magnetic intensity vector and magnetization vector. In addition, ρ and η are the fluid's density and viscosity. μ_0 is the magnetic permeability of a vacuum.

For Shliomis model, the effect of a homogeneous magnetic field on the viscosity of FFs whose solid particles possess intrinsic magnetic moments is investigated. The applied magnetic field impedes particle rotation in a vortical liquid flow, resulting in an increment of the effective viscosity. The characteristic of Shliomis model is that the effect of rotation of magnetic particles, their magnetic moments and the volume concentration were included. According to Shliomis model, assuming steady flow, neglecting inertia and the second derivative of the internal angular momentum **S**, the flow can be presented as follows[37]:

$$-\nabla p + \mu_0(\mathbf{M}\cdot\nabla)\mathbf{H} + \eta\nabla^2\mathbf{v} + \frac{1}{2\tau_S}\nabla\times(\mathbf{S} - I\mathbf{\Omega}) = 0 \tag{5.1}$$

$$\mathbf{S} = I\mathbf{\Omega} + \mu_0\tau_S(\mathbf{M}\times\mathbf{H}) \tag{5.2}$$

$$\mathbf{M} = \mathbf{M_0}\frac{\mathbf{H}}{H} + \frac{\tau_B}{I}(\mathbf{S}\times\mathbf{M}) = 0 \tag{5.3}$$

where I is the sum of inertia moments of the particles per unit volume and $\Omega = (1/2)\nabla\times\mathbf{v}$. τ_S and τ_B are the magnetic moment relaxation time and Brownian relaxation time. $\mathbf{M_0}$ is the equilibrium magnetization vector and H is the magnitude of **H**.

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Jenkins presented the isothermal static equilibrium theory for FFs in some detail. In determining the system of equations governing the state, he obtained integrals of the linear momentum equations and identified the magnetic energy density function for FFs. A continuum theory was proposed, and for incompressible FFs, the equations of the Jenkins model for steady flow neglecting inertia term can be written as follows[39]:

$$-\nabla p + \eta\nabla^2\mathbf{v} + \mu_0(\mathbf{M}\cdot\nabla)\mathbf{H} + \rho\alpha^2\nabla\times\left(\frac{\mathbf{M}}{M}\times\mathbf{M}^*\right) = 0 \tag{6.1}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{6.2}$$

$$\nabla\times\mathbf{H} = 0 \tag{6.3}$$

$$\nabla\cdot(\mathbf{H} + 4\pi\mathbf{M}) = 0 \tag{6.4}$$

$$\mathbf{M} = \chi H \tag{6.5}$$

$$\mathbf{M}^* = \frac{1}{2}(\nabla\times\mathbf{v})\times\mathbf{M} \tag{6.6}$$

where α is material constant, χ is the magnetic susceptibility of the fluid and $\mathbf{M}^*$ is co-rotational derivative of **M**.

In this model, the component of the spin that is parallel to the magnetization is ignored. Compared with Neuringer–Rosensweig model, the volume force density in Jenkins model due to the self field is distinguished from the external body force, which includes, for example the volume force due to an external magnetic field.

Using the aforementioned models as the basis, the squeeze film performances for two approaching surfaces with various physical configurations were investigated. In the following, special attention was paid on FFs film lubrication based on the three models.

Neuringer–Rosensweig model

Neuringer and Rosensweig performed a substantial fraction of the pioneering effort in this field. Neuringer–Rosensweig model presents the fundamental hydrodynamic equations describing the flow of FFs. They concentrated upon the influence of the magnetic body force on a paramagnetic fluid characterised by a symmetric Newtonian stress tensor and discussed thermo-mechanical phenomena in this model. According to their theory, the equations of fluid motion differ from ordinary Navier–Stokes equation owing to the presence of magnetic force.[40] It was pointed that the vorticity may be generated by thermo-magnetic interaction, which led to the development of augmented Bernoulli relationships.[36] In most FFs-bearing studies, the governing equations for FFs were adopted from the flow model of Neuringer–Rosensweig.

The earlier work has been done by Tarapov,[41] who considered bearings lubricated with FFs in a non-uniform magnetic field. He proposed a displaced infinitely long current-carrying-wire model. It gave field variation across the circumferential direction, and the results showed that the bearing load capacity increases in the magnetic field due to magnetic levitation force acting on a rotating shaft.

Subsequently, Walker and Buckmaster[42] analysed the dynamic behaviours of FFs when considering a thrust bearing. The FFs, as lubricant, was confined by an applied field to the gap between two plates. The steady flow of a viscous, incompressible and magnetizable fluid was governed by the equations

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suggested by Neuringer–Rosensweig model. The typical parameters of plate and FFs were chosen, which led to three assumptions: (i) the magnetic field in the plates was negligible, (ii) the magnetization in the FFs was negligible compared with the external field and (iii) the FFs was saturated. It was demonstrated that ferrohydrodynamic thrust bearings with magnetic plates are stiff, i.e. changes in axial load produce relatively small variations in gap width.

Tipei[43] deduced a pressure differential equation, equivalent to Reynolds equation, and applied it to study short bearings. The FFs was treated as Newtonian fluid. Here, it was shown that the FFs lubricant improves the performance of the bearing system and bearing stability and stiffness. In addition, the FFs lubrication can reduce wear.

Verma[44] considered the squeeze film lubrication of FFs between two approaching rectangular surfaces in the presence of an externally applied magnetic field oblique to the lower surface. The explicit solutions for the velocity, pressure and load capacity were discussed. The performance of the FFs-based squeeze film was definitely enhanced, and the time for the upper plate to come down was found to be longer than the viscous squeeze film.

Chi *et al.*[45] proposed a new type of FFs-lubricated journal bearing with three pads and found its performance much better than that of a conventionally lubricated bearing both theoretically and experimentally. Moreover, the bearing operated without leakage and any feed system.

In spite of these efforts, the squeeze film performances for two approaching surfaces with various physical configurations have also been examined extensively by Shat and his colleagues, for example, the step plates,[46] the annular plate[47] and the curved circular plates.[48,49] Representative sketch map is shown in Figures 4 and 5. According to the expressions deduced, the authors found that the pressure and load capacity both increase with increasing magnetization parameter. In addition, for the step bearing system, the load capacity became more when the length of the first step was taken less,[46] while the effects due to magnetization were independent of the curvature of the upper disc for the curved circular plates.[48]

A newly designed double porous layered axially undefined journal bearing including combined effects of anisotropic permeability, slip velocity and squeeze velocity was studied by Shah and Patel.[50] The bearing is lubricated with water-based FFs and the basic equations are governed by the flow of Neuringer–Rosensweig model. The values of dimensionless load capacity were computed using FFs as lubricant and it increased up to 206% compared with conventional lubricant.

Based upon the FFs model of Neuringer–Rosensweig, Osman *et al.*[51] have considered an FFs journal bearing subjected to cavitation condition and shown theoretically that a magnetic field decreases

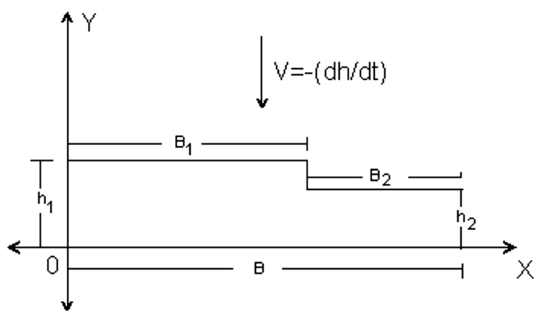

Figure 4. Configuration of step plates.[46] The bearing with two flat surfaces parallel to the *x*-axis of the film thickness.

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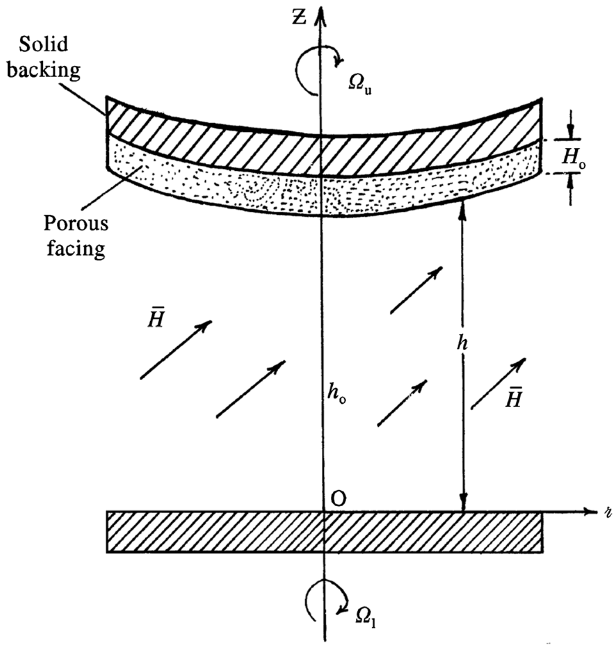

Figure 5. Configuration of curved circular plates.[49] The bearing with curved porous rotating circular plates in the presence of an external magnetic field oblique to the lower plate.

the leakage rate. They explained it by intense inward flows of an FFs lubricant towards cavitation region, but the nature of this effect is not still completely clear. It was also found that bearing characteristics can be significantly improved when the magnetic influences are comparable with the hydrodynamic effects. Recently, combined with Neuringer–Rosensweig and the Stokes micro-continuum theory,[52] the coupled effects of non-Newtonian couple stresses and FFs on the journal-bearing performances were investigated by Nada and Osman.[53] It was concluded that the performances of journal bearings lubricated by non-Newtonian FFs can be improved. Similar result[54] was reported that the presence of couple stress lubricant can enhance the load capacity and reduce the coefficient of friction. Later on, the thermal effect was taken into account by Nada *et al.*[55] to investigate hydrodynamic journal bearings lubricated by FFs with couple stresses. It showed that thermal aspects affect the lubricant viscosity, and consequently, lower hydrodynamic pressure, reduced load capacity, less frictional force and relatively lower rates of side leakage are obtained compared with isothermal condition. Both the pressure and load capacity increase with the increase of the couple stress parameter. The authors concluded that fluids with couple stress are better than Newtonian fluids. However, the effect of surface roughness is not considered.

The combined effects of stochastic surface roughness and a magnetic field on short journal bearings[56] and long journal bearings[57] lubricated with FFs were examined by Hsu *et al.* Figure 6 depicts the configuration of a short bearing with lubrication of the interior bearings provided by FFs. Compared with bearing with smooth surfaces, they observed that the introduction of longitudinal roughness for short journal bearing and transverse for the long is conducive to enhance the film pressure and load capacity.

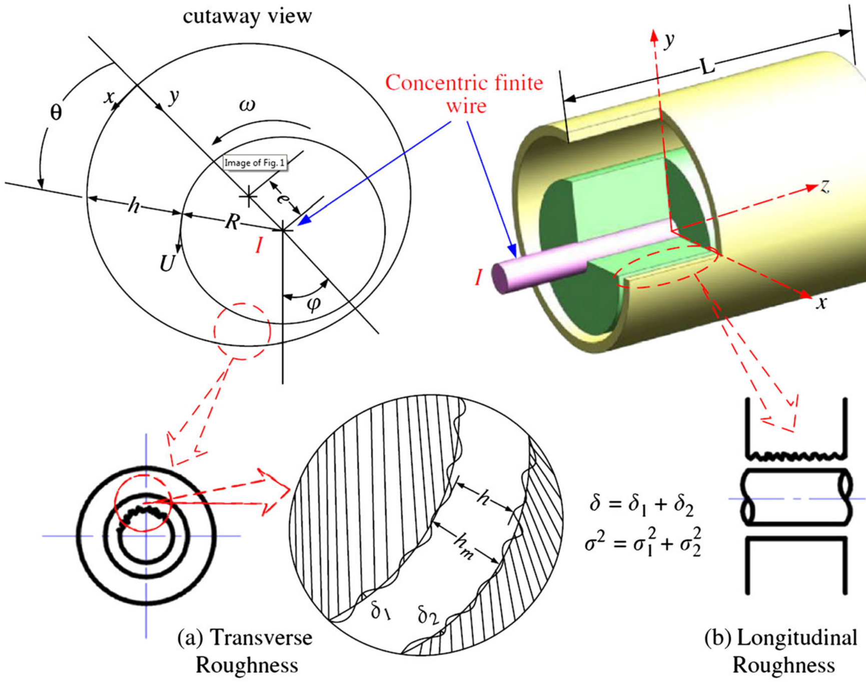

Figure 6. Configuration of a short bearing.[56] Short bearing with a journal radius R rotating at angular speed ω.

Numerous researches using Neuringer–Rosensweig model have been carried out theoretically. A short conclusion can be summarised as follows: an FFs lubricant increases the pressure as well as load capacity of the bearings, improves bearing stability and stiffness and reduces wear noise and maintenance costs.

Shliomis model

Shliomis[37] proposed an FFs flow model in which the effect of rotation of magnetic particles, their magnetic moments and the volume concentration were included. Consideration was given to various effects caused by rotation of the particle, anisotropy of the viscosity, entrainment of the suspension by a rotating field and dependence of the kinetic coefficients on the field intensity. A great number of theoretical investigations about FFs lubrication are based on Shliomis model. The list, which is no exhaustive, includes applications and advances in the FFs lubrication based on Shliomis model.

Shukla and Kumar[58] are the earlier ones employing Shliomis model. A generalised form of Reynolds' equation governing the FFs film pressure in the presence of a transverse magnetic field was derived by considering particles rotation. The equation was illustrated in cases of squeeze film and slider bearings. To simplify the analysis, several assumptions were made as mentioned by Walker and Buckmaster.[42] The authors came to a conclusion that the Brownian motion of the particles together with rotation of the magnetic moments within the particles produces rotational viscosity, which can support more loads.

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Chandra *et al.*[59] studied a journal bearing considering cavitation. They assumed that the magnetization vector is not parallel to the magnetic field vector. It was found that qualitative behaviour of the bearing characteristics remains similar to that in the case of nonmagnetic fluid. After that, Sinha *et al.*[60] continued to analyse the cylindrical rollers under combined rolling and normal motion with cavitation. The configuration of the bearing was shown in Figure 7. The advanced prospects of this work are that it relaxes the assumptions of the magnetization vector being parallel to the magnetic field and the rotation of magnetic particles has also been accounted for. It was found that the application of the magnetic field to the FFs lubrication increases the load capacity without affecting the point of cavitation and the performance of rollers operating with a thinner oil film can be enhanced. However, the pressure and load capacity for FFs remain similar to that of a Newtonian fluid, and the impact of the FFs parameters on the cavitation point is negligible.

Recently, Singh *et al.*[61] studied the effect of FFs on the dynamic (damping and stiffness) characteristics of curved slider bearings using Shliomis model, which accounted for the rotation of magnetic particles, their magnetic moments and the volume concentration in the fluid. Similar results were observed that the effect of rotation and increased concentration of magnetic particles improves the stiffness and damping capacities of the bearings. Besides, compared with the conventional lubricants, the dynamic stiffness and damping coefficients are higher for FFs even if the magnetic field is absent.

The performances of a Shliomis model-based FFs lubrication of a plane inclined slider bearing with velocity slip were analysed by Patel and Deheri.[62] It was realised that the magnetization could also reduce the adverse effect of surface roughness up to some extent when suitable values of slip parameter were in place. Then Patel and Deheri reported the FFs lubrication of a squeeze film in rotating rough porous curved circular plates[63] and rough curved annular plates.[64] The stochastic modelling of transverse roughness was employed in the two literatures. The associated Reynolds-type equation was solved to obtain the pressure distribution, which leads to the calculation of the load capacity. Figure 8 presents the configuration of curved annular bearing system. It was pointed out that the adverse effect of transverse roughness can be overcome by the positive effect of FFs lubrication, and compared with Neuringer–Rosensweig model, a constant magnetic field did enhance the characteristics in Shliomis model. Further, the roughness must be given due consideration while designing the bearing system.

Based upon the hydrodynamic flow model of Shliomis, a study concerning the rotation of magnetic particles in squeeze film was presented for the curved annular plates by Shah and Bhat.[65] The pressure

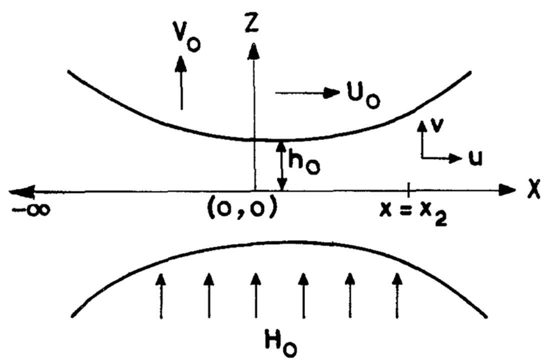

Figure 7. Configuration of cylindrical rollers.[60] The bearing with axisymmetric flow of ferrofluids between rollers moved with equal uniform tangential velocity (U_0) and normal velocity (V_0).

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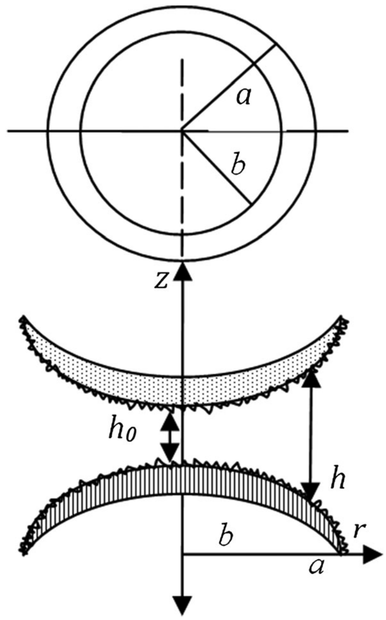

Figure 8. Configuration of the curved annular plates bearing system.[64] The bearing consists of two annular plates with each of inside radius b and outside radius a ($a > b$).

equation derived in the work differs from that used by Shukla and Kumar,[58] which is deduced under the assumptions of the saturated FFs and neglected magnetic moment relaxation time. It figured out that the load capacity and approaching time of squeeze films can be enhanced by increasing the volume concentration of FFs and the intensity of external magnetic fields. Recently, Shah *et al.*[66] discussed the various designed slider bearings with inclined pad stator, exponential pad stator, secant pad stator, convex pad stator and parallel pad stator and compared for dimensionless load capacity using FFs lubricant under a constant transverse magnetic field with and without the effect of squeeze velocity. It was concluded that the load capacity of all bearings remains constant with the increase of Langevin's parameter, whereas it has an increasing tendency with the increase of volume concentration of the particles. Unfortunately, the influence of fluid inertia forces was not mentioned in their researches.

Because of the increasing fluid velocity, the effects of fluid inertia forces need to be included in the study of FFs-lubricated squeeze films. In this respect, lots of work based on Shliomis model has been done by Lin and his colleagues.[67–69] According to their results, the effects of fluid inertia forces were found to yield an increase in the load capacity and to prolong the approaching time for the FFs sphere-plate system in the presence of transverse magnetic field. Furthermore, for the squeezing circular plates is a special case of squeeze film between conical plates, the squeeze film characteristics of FFs-based conical plates including fluid inertia force is of interest. Compared with the non-inertia case, FFs-based conical plates have a better performance.

Combined with the Shliomis model and the micro-continuum theory of Stokes,[52] the effects of non-Newtonian couple stress FFs on the steady-state performance of long journal bearings were also assessed by Lin *et al.*[70] Analytical solutions for bearing performances were obtained from the non-Newtonian FFs Reynolds-type equation. Compared with the Newtonian fluid case, the effects of

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non-Newtonian FFs under external fields provided increased values of the zero pressure-gradient angle and the load capacity, and resulted in decreased values of the friction parameter especially for a larger non-Newtonian couple stress parameter and magnetic Langevin's parameter. Similar study was reported[71] considering the effects of viscosity-pressure dependency on the non-Newtonian squeeze film in parallel circular plates. The work revealed that the influences of viscosity-pressure dependency raised the load capacity.

Followed by Shliomis, the magnetic particles in carrier liquid under external field can be relaxed by the rotations of particles (Brownian relaxation) and internal magnetization vector inside the particle (Néel relaxation). A general conclusion can be drawn that Brownian together with Néel motion produces rotational viscosity that supports a higher load capacity.

Jenkins model

The hydrodynamics of FFs were also discussed by Jenkins. In 1971, Jenkins employed a simple continuum model for a paramagnetic fluid to analyse the shearing flow and parallel flow through a pipe and examined the possibility of maintaining a steady circular flow in a circular cylinder by rotating a magnetic field.[72] After that, he pursued the subject including the local magnetization as an extra hydrodynamical variable independent of the usual variables such as pressure, density, temperature and velocity.[38] During the analysis, the isothermal static equilibrium theory was employed, and the integrals of the linear momentum equations and the magnetic energy density function for FFs were obtained. Awareness on Jenkins flow model has been increased in recent years.

Ram and Verma[73] studied porous inclined slider bearing with FFs lubricant flowing according to the Jenkins flow behaviour. The co-rotational derivative of the magnetization vector was parallel to the magnetic field vector. The load capacity of the bearing had been numerically compared by taking different values of the FFs parameters. During computation, all field quantities were in steady state, and the self-field created by the magnetization of the fluid was neglected. They noticed that the value of load capacity increases considerably when using all the terms of Jenkins model. Yet, the model is more realistic for the consideration of the bearing material constant with unit of angular momentum per unit mass of the fluid per unit field strength.

Shah and Bhat did a huge amount of work in FFs lubrication whose flow is governed by Jenkins flow behaviour. In literature,[74] they derived a Reynolds-type equation in a squeeze film between two circular plates considering combined effects of rotation of the plates, anisotropic permeability in the porous matrix and slip velocity at the interface of porous matrix and film region. It was found that, the load capacity increases with increasing values of rotation parameters and it attains a maximum for a value of material parameter near 1. However, for the Neuringer–Rosensweig model, the load capacity decreases slowly with increasing values of rotation parameters. In addition, anisotropic permeability of the porous face can be used to increase the load capacity of a parallel plate squeeze film porous bearing. In the same year, a porous exponential slider bearing was also studied by Shah and Bhat,[75] considering slip velocity at the porous interface. It was observed that the load capacity as well as the friction decreased when the slip parameter increased. The pressure in centre position was not affected significantly by the slip parameter, but it shifted towards the bearing inlet for large values of material parameter.

After that, a deeper analysis of a slider bearing with its stator having a circular convex pad surface was done.[76] The results suggested that the load capacity increased with the film thickness ratio. The friction force on the slider decreased with increasing film thickness ratio while the pressure in the

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position centre shifted towards the outlet. Later on, Shah and Bhat[77] theoretically analysed the effects of slip velocity and an FFs lubricant characterised by a material parameter on a parallel plate porous slider bearing. They made an important discovery that the parallel plate slider bearing lubricated with FFs can support a load, while a conventional lubricant cannot.

Based on Jenkins model, Patel and Deheri[78] studied the performance of a rough porous parallel plate squeeze film slider bearing under the presence of an FFs lubricant considering velocity slip. The investigation suggested that the roughness must be given due consideration while designing the bearing system even if the magnetic field strength was suitably chosen. They also confirmed that a parallel plate slider bearing can support a load with FFs lubrication, as mentioned by Shah and Bhat.[77] Afterwards, an FFs-lubricated rough porous inclined slider bearing[79] and an infinitely long rough journal bearing[80] were examined subsequently. The geometry and configuration are presented in Figures 9 and 10. It was established that the two bearings suffer, owing to transverse surface roughness.

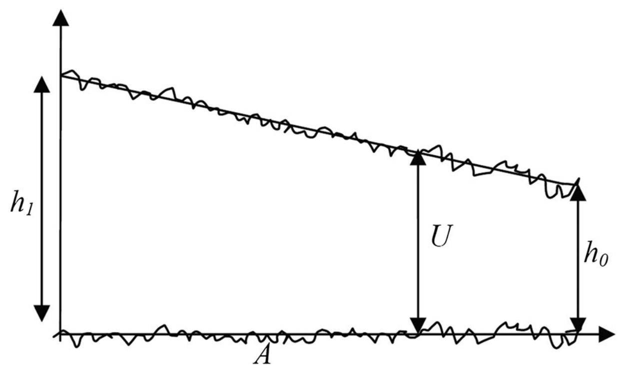

Figure 9. Configuration of the porous inclined bearing system.[79] A rough, porous inclined slider bearing.

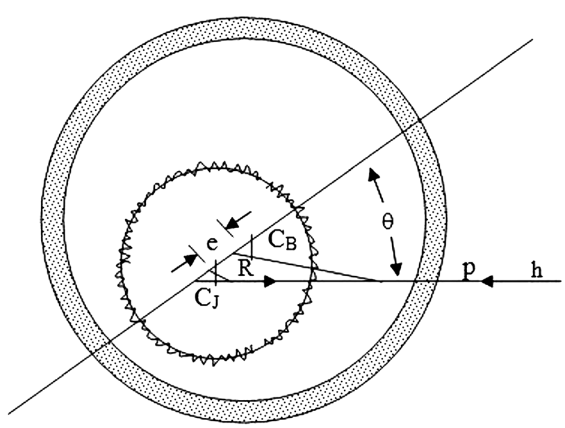

Figure 10. Configuration of the long rough journal bearing system.[80] The bearing with a journal radius *R* inside and the gap is filled with ferrofluids under an external magnetic field of strength.

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The performance of the bearing system can be improved by suitably choosing the magnetization parameter and slip coefficient in the case of negatively skewed roughness.

An endeavour was made by Shukla and Deheri[81] to discuss the performance of a transversely rough porous circular convex pad slider bearing in the presence of an FFs lubricant. With the aid of suitable boundary conditions, the associated stochastically averaged Reynolds' equation was solved to obtain the expression for pressure distribution. The authors pointed out that a limited scope of the magnetization for compensating the adverse effect of porosity, roughness and material parameter existed. It was also interestingly noted that the film thickness ratio turns in a marginally better performance as compared with most of recent studies.

Summary of the three models

As mentioned earlier, Neuringer–Rosensweig model provides the fundamental hydrodynamic equations describing the flow of FFs, and it focused on the influence of the magnetic body force on a paramagnetic fluid characterised. However, in a uniform magnetic field, magnetic pressure cannot be generated in the Neuringer–Rosensweig model, while the pressure, which affects the bearing characteristics, could be formed in the Shliomis model owing to the rotational viscosity. The load capacity and squeeze time are more in the case of a non-uniform magnetic field than in the case of a uniform magnetic field owing to the effect of magnetization in the former case.[82]

As pointed by Shliomis,[37] the magnetic particles in FFs can relax by the rotation of particles in the fluid (Brownian relaxation) and by rotation of magnetic moment within the particles. Considering the Shliomis model, when the FFs is saturated, then the saturation magnetization of FFs is independent of the applied magnetic field and the load capacity of squeeze film can only be effected by Brownian relaxation time since the magnetic moment relaxation time is negligible.[58]

For consideration of material parameter in Jenkins model, the nature of the results obtained for the bearing characteristics might be more significant than that of the Neuringer–Rosensweig model. In a theoretical study, material constant may take the three aspects of the rotation of the fluid, fluid mass and the field strength, which are usually considered during the bearing design process. In addition, as mentioned by Shaha and Bhat,[83] anisotropy of porous matrix has no effect on the bearing characteristics when considering one-dimensional problems of flow with Neuringer–Rosensweig model, while Jenkins model does.

EXPERIMENTAL STUDIES ON FERROFLUIDS LUBRICATION

Up to date, most investigations of FFs lubrication have been examined by theoretical analysis and only a few by experimental studies. Admittedly, the experimental evaluation of FFs lubrication property is very important, and related results will also help to design FFs-lubricated bearing assemblies. In the following sections, experimental studies on FFs lubrication have been summarised in details.

Four-ball machine tests

The four-ball machine is known as an often used device to evaluate the lubricant. The lubrication properties of FFs containing $Mn_{0.78}Zn_{0.22}Fe_2O_4$ and Fe_3O_4 nanoparticles were examined by Wang *et al.*[84] and Huang *et al.*,[85] respectively, using a four ball tester. Both experimental results showed clearly that

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the FFs have much better friction reduction and anti-wear abilities than the base oil (Figure 11). At the same time, the maximum non-seized load has been increased significantly compared with the base oil.

Deysarkar *et al.*[24] presented the effect of magnetic field on friction characteristics of the FFs using a modified machine. The four balls and cup holder were machined from non-magnetic material, and a special constructed magnet was placed around the sample holder. Interesting results were observed that the existence of magnet demonstrates no obvious effect on friction and wear properties. In addition, compared with the fluid in the absence of a magnet, the presence of a magnetic field resulted in a more uniform wear pattern on the scars of the balls. Similar experiments were performed by Wang *et al.*[86] However, their observation of better friction-reducing and anti-wear abilities with the present of magnetic field is in contradiction to those of Deysarkar *et al.*

Bearing lubrication tests

There is no doubt that bearing lubrication is the most important application for FFs. Miyake and Takahashi[87] carried out friction and wear tests on cylindrical specimens constructed of permanent magnets under the lubrication of FFs to develop a thrust bearing that could be used in clean circumstances. They declared that the boundary lubricating effect of FFs can reduce the friction and wear under low-velocity condition. The hydrodynamic lubricating effect, on the contrary, shows friction reduction in the high-velocity conditions.

In 2009, Huang *et al.*[88] have focused on thrust bearings lubricated with FFs. The magnetic field was produced by 8 cylindrical NdFeB magnets, with the size of $\phi 4.0\,mm \times 4.0\,mm$, distributed uniformly on a cylinder surface. The images of the cylinder surface before and after covering with FFs are shown in Figure 12. The early experimental research has verified that the FFs can be used to improve the static loading capacity and friction characteristics of bearings under an external magnetic field. Figure 13 shows the optical microscope images of worn scar lubricated with carrier liquid and FFs. After that, the tribological properties of FFs under different magnet distributions were examined experimentally.[89] The four samples covered with FFs are shown in Figure 14. Columniform NdFeB magnets with different sizes (magnetic property: 35MGOe) were used, while the area ratio of the permanent magnets on each cylinder surface, as well as the total volume in each cylinder, was the same. In Figure 15, as

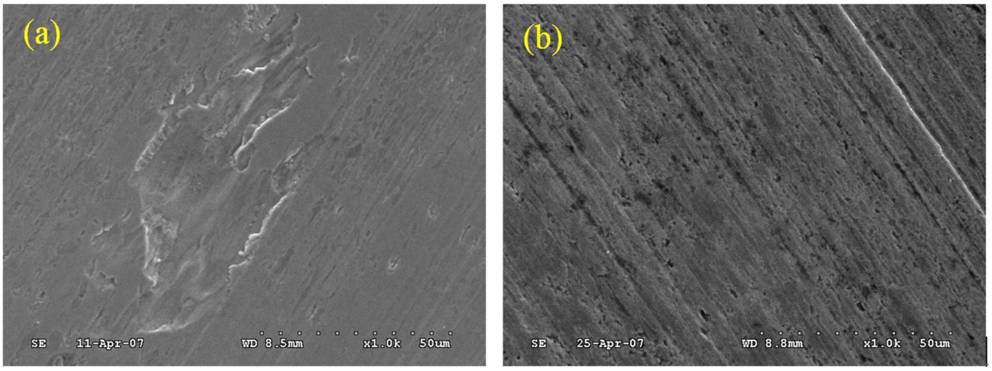

Figure 11. Scanning electron microscopy morphologies of the worn surfaces: (a) lubricated with base oil and (b) lubricated with ferrofluids.[84]

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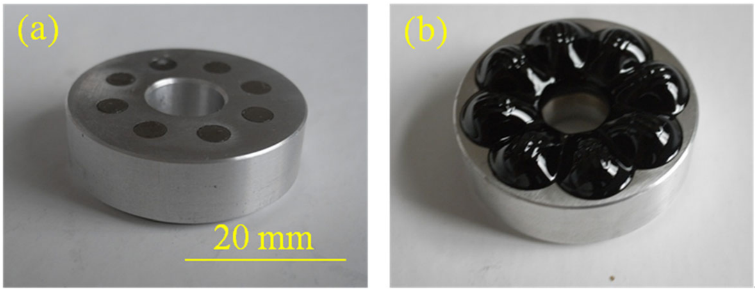

Figure 12. (a) Cylinder surface distribute eight cylindrical NdFeB magnets and (b) cylinder surface covered with ferrofluids.[88]

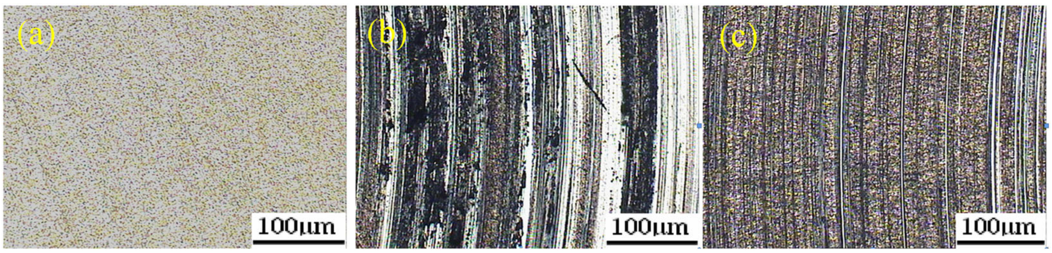

Figure 13. Optical microscope images of worn scar lubricated with carrier liquid and ferrofluids: (a) before test, (b) lubricated with carrier liquid and (c) lubricated with ferrofluids.[88]

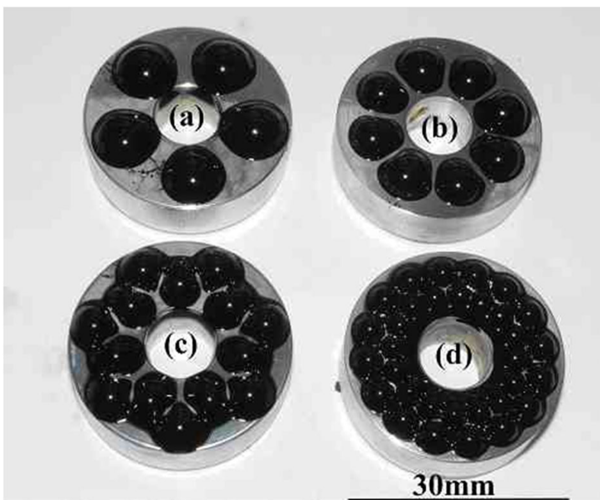

Figure 14. Cylinders covered with ferrofluids: (a) five magnets (φ5.0 mm × 4.0 mm), (b) eight magnets (φ4.0 mm × 4.0 mm), (c) 14 magnets (φ3.0 mm × 4.0 mm) and (d) 32 magnets (φ2.0 mm × 4.0 mm).[89]

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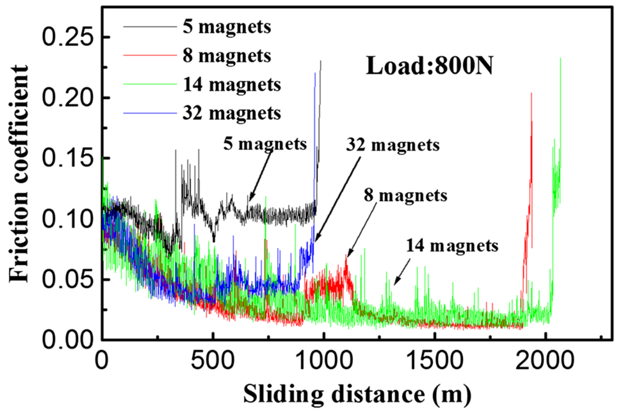

Figure 15. Evolution of the friction coefficient on the four cylinders lubricated with ferrofluids.[89]

can be seen that the magnetic field distributions on the rubbing surface has a significant influence on the lubrication properties.

To reduce the size of the bulk magnet, a novel magnetic surface was designed for FFs lubrication.[90,91] Micro dimple patterns were first fabricated on a disk surface and the permanent magnet films then were electrodeposited into these dimples. Figure 16 shows the magnetic surfaces before and after covered with FFs. It can be seen that an ordered pattern of fluid bumps is formed on disk's surface due to magnetic interaction between magnetic arrayed film and FFs. The static contact angel on normal and magnetic film was measured. And it was found that the angles increase with the increasing film thickness (Figure 17), which is due to the enhanced magnetic pressure in fluid per unit of volume.[92] The tribological performances of the magnetic arrayed surface were evaluated under different speed-load conditions.[93–95] The overall result of these tests is that compared with normal surface, the magnetic surfaces present friction reduction at higher sliding speed.

Ochoński performed lots of research work on the design of FFs lubricated bearings including some new designs of FFs exclusion seals for rolling bearings[96] and sliding bearings lubrication.[97] The main advantages of these bearings over conventional bearings were introduced, such as extremely low non-repetitive run-out, good damping and quietness of operation, maintenance free service and high reliability.

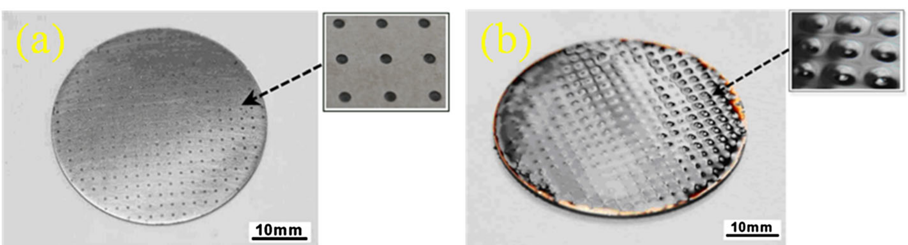

Figure 16. (a) Disk with magnetic arrayed film; (b) Disk with magnetic surface covered with ferrofluids.[90]

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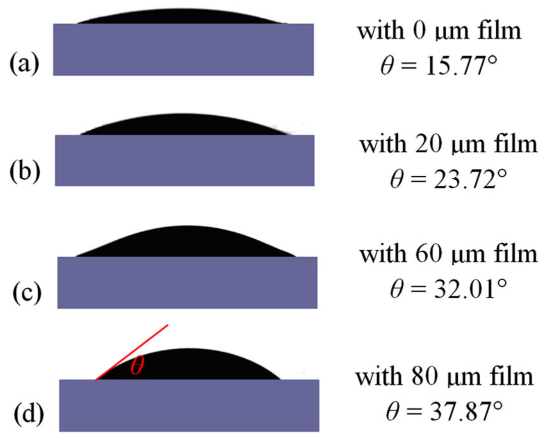

Figure 17. The droplet shapes of ferrofluids on normal and magnet arrayed surface.[92]

Urreta *et al.*[98] set up a test bench for plain journal bearings lubricated with FFs and magnetorheological fluids (MR). Theoretical analysis was carried out with numerical solutions of Reynolds equation, based on apparent viscosity modulation for FFs and Bingham model for MR. It was pointed out that FFs cannot be used as tunable lubricant for its low rheological change. However, MR achieves good performance as active fluids in the bearing, and it could be a good solution to increase the operation range of hydrodynamic bearings.

Wang *et al.*[99] provided a theoretical and experimental analysis of oil-film bearing lubrication using FFs. A solenoid was developed to provide magnetic field, and the effects of oil-film temperature, oil-film pressure and magnetic intensity on FFs viscosity were discussed. It showed that the viscosity can be controlled to expand the load-carrying scope at different regions, thus improving load capacity of bearing.

Others

Uhlmann *et al.*[25] investigated the tribological behaviour of FFs under boundary conditions using a modified ball on disk tester. This research figured out that FFs can reduce the wear under boundary condition. The authors also pointed out that FFs can be used as lubricants in gears, plain and roller bearings with an appropriate static magnetic field for the fixation of the lubricants.

Andablo-Reyes *et al.*[100] developed a method of an FFs lubrication to control starvation in lubricated contacts. Ball-plate contact is experimentally studied under sliding-rolling conditions. It showed that FFs can be used as a smart lubricant to limit starvation in a sliding–rolling contact where the amount of lubricant is restricted. It is also proposed that the higher viscosity and the magnetic field strength were required to prevent starvation. Another study also by Andablo-Reyes *et al.*[101] was carried out in full film hydrodynamic regime under the presence of external magnetic field gradients. By using FFs in mechanical contacts, the frictional behaviour can be actively controlled by the application of magnetic fields.

Shahrivar *et al.*[102] compared the lubricating properties of FFs and MR in the absence/presence of external magnetic field using the ball-on-three-plates geometry and stainless steel-steel point contacts. Figure 18 shows the friction coefficient as a function of sliding speed. In the absence of magnetic field, the FFs operates in the boundary lubrication regime at low sliding speeds, while the MR fluid essentially operates during the whole range of speeds. And, surprise, the magnetic field has little influence

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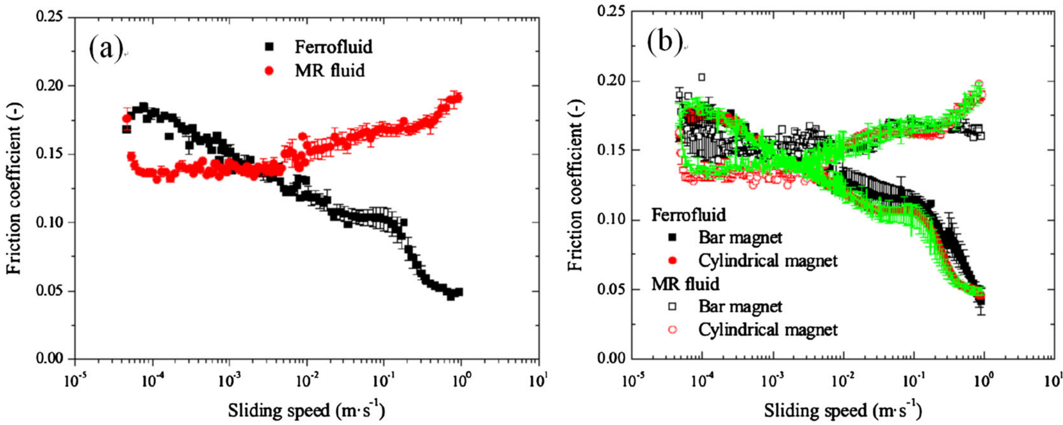

Figure 18. Friction coefficient as a function of sliding speed lubricated with the ferrofluids and magnetorheological fluids: (a) without magnetic field and (b) with magnetic field.[102]

on the frictional behaviour of both lubricants (Figure 18b) for the viscosity of the magnetic lubricants at high shear rate does not significantly depend on the magnetic field strength. An interesting finding is that the particle concentration, over the particle size, dominates the lubrication properties of such magnetic colloids. Recently, Shahrivar *et al.*[103] investigated the frictional properties of Newtonian fluids and FFs in compliant point contacts. Special emphasis is given to the full-film isoviscous elastic lubrication regime. Under appropriate conditions, a friction reduction is observed by simply displacing the magnetic field distribution in the flow direction towards the inlet of the contact.

Summary of the experiments

It seems that using FFs as a lubricant is significantly effective in reducing friction and wear properties since it can be retained at the desired location by introducing an external force. Thus, it can be an effective way to control starvation in contacts. Compared with carrier liquid, obvious advantages of FFs lubrication appeared according to the four-ball machine tests. In bearing lubrication aspect, different kinds of magnetic surfaces and the corresponding lubrication properties are suggested. Using the magnet arrayed surface seems to have a good application prospect.

Since FFs can absorbed by external field, FFs lubricated bearings will be the best candidate for space microgravity environment. In elastic contact area, FFs can also be employed for its controllable viscosity and rheological behaviours. Recently, a new kind of FFs using ionic liquid as carried liquid was reported. It supplies a possibility to study the FFs lubrication properties under magnetic and electric fields due to the good electrical conductivity of ionic liquid. With the development of science and technology, FFs lubrication will have a huge range of applications.

CONCLUSIONS

Since a stable FFs obtained in the mid-1960s, this kind of smart material is commercially available and has been applied extensively in the field of engineering technology. For the prominent characters of

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being controlled and located at some preferred places in the presence of an external magnetic field, FFs has been successfully employed as lubricants in various bearings system.

This paper presents an overview of the recent developments in the study of FFs lubrication, including FFs hydrodynamic bearing lubrication theories and experimental lubrication evaluations. In theoretical studies aspect, three flow models of Neuringer–Rosensweig, Shliomis and Jenkins were taken into account. It is found that in most early studies, the governing equations for FFs were taken from the flow model of Neuringer–Rosensweig. Later, Shliomis model has been considered gradually to study bearing characteristics in various situations. At the same time, Jenkins model was also caused extensive concern. It should be noted that some theoretical results are not unanimous according to the three flow models since simplified assumptions in each model have been made. In addition, only a few experimental studies of FFs lubrication have been reported till now. The existing results indicated that FFs under external magnetic field has much better friction reduction and anti-wear abilities over traditional lubricants.

Though lots of work has been made on FFs lubrication theoretically, most of the results are not justified by experiments. Effort is still needed to provide references for the future design of FFs-lubricated bearings.

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